Deterministic directed transport of inertial particles in a flashing ratchet potential

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Deterministic directed transport of inertia particles in a periodically on-off ratchet potential is investigated. We find that the directed transport can be induced by a finite inertia; i.e., in the overdamped case, no directed motion of the system can be observed. It is shown that a critical threshold of the ratchet asymmetry is required for the system to achieve a net current. Directed transport can be greatly enhanced when the coupling strength of particles is increased. An appropriate match of the coupling, the flashing period, and the damping can give rise to the best efficiency of transport. The commensurate effect on the directed transport, which originates from the spatial competition between the period of the potential and the static length of the coupling, is analyzed.

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I. INTRODUCTION

In recent years, much effort has been devoted to understand the nonequilibrium mechanism of generating net currents by the rectification of thermal fluctuations in the presence of certain drivings with temporally, spatially, and statistically zero mean [1–3,5]. The fundamental property of detailed balance excludes the possibility of nonzero net currents at the thermodynamic equilibrium [6]. On the other hand, directed transport (DT) has been observed in the absence of any macroscopic gradient of forces, if only the substrate potential exhibits the spatial asymmetry and detailed balance is broken. These explorations helped us to get a deeper understanding of the mechanism of many phenomena in molecular motors [2], flux dynamics in superconductors [7], Josephson junctions arrays, ladders, and lines [8], transport in quantum dots [9], nanodevice design, particle separator, and solid surface treatments [10].

Deterministic ratchets received much attention recently in revealing the dynamical mechanism of directed transport [11,12]. It has been shown that periodic and chaotic forces [12] can lead to directed transport, even in the absence of thermal fluctuations. In studies of bimolecular motors, it has been shown that intrinsic degrees of freedom are essential for net directional motion to occur. For example, molecular motors in muscles have linear structures, which consist of many parts [13]. The proteins of the kinesin superfamily are found to be composed of two globular "heads." The kinesin direction of motion along microtubules could be reversed by adjusting the architecture of a small domain of the protein called the neck region [14]. These experiments reveal the fact that the DT sometimes depends less on the fluctuation environment, whileas intrinsic structures (architectures) and symmetry properties of the system may play a more important role in producing a net current [15].

In physics, a valuable topic is the connection between the macroscopic directed transport and the microscopic dynamics. In most of these studies, it was found that the inertia effect plays an important role in governing the transport behavior [16]. With a finite inertia, the dynamics is allowed to become more complicated, exhibiting both regular and chaotic behaviors. Therefore it is still a challenge to study the finite-inertia effect on the ratcheting mechanism in the absence of thermal and nonthermal forcings.

A significant topic in studies of directed transport is the collective phenomena induced by mutual interactions. Motivated by numerous biological and physical systems, collective transport of spatially extended systems such as diffusions, spatiotemporal pattern dynamics and waves, and stochastic resonances has been extensively explored [4,5]. In fact, mutual interactions among different degrees of freedom introduce additional spatial competitions. This competition can bring novel and complicated dynamics of transport, which have been be ubiquitously observed in many practical situations and experiments. In recent years there have been a number of explorations on directed transport in coupled systems—e.g., the rocking overdamped ratchet lattice with harmonic couplings [17], ratchet motion of particles with hard-core interactions [18], asymmetrically coupled lattice in symmetric potentials without external forces [19], ratchet motion by breaking the spatiotemporal symmetries [20], and so on (for a comprehensive list of references, see [1,5]).

Until now there have been very few discussions on the collective directed effect of spatiotemporal systems in the absence of a noise background. In this paper it is our task to investigate the inertia-induced directed transport of an elastically coupled chain of particles in an on-off ratchet potential. We focus on the noiseless and damped case. To our knowledge, no one has studied the directed motion of an inertia particle in an on-off ratchet potential in the absence of external forces. We find that a net current can be found when the asymmetry of the ratchet potential exceeds a critical value. The coupling of particles may enhance the transport current when the coupling strength overcomes a threshold. The flashing period of the ratchet potential has an optimal value for the highest efficiency of the transport. We also find that a net current exists only in a finite regime of the damp-

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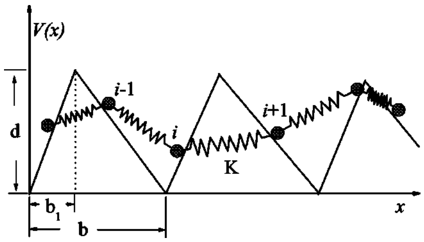

FIG. 1. A schematic plot of the coupled sawtooth ratchet.

ing; i.e., for large dampings directed current may disappear. For a single particle, negative current can be observed, whileas this reversal is eliminated in coupled lattices. The commensurate effect of direct transport is also discussed.

II. MODEL

The model system we choose in this work is a chain of particles in a flashing ratchet potential. For simplicity, we consider the nearest-neighbor harmonic coupling among particles. The equations of motion of N particles can be written as

$$m\ddot{x}_i = - m\gamma\dot{x}_i - V'(x_i,t) + K(x_{i+1} - 2x_i + x_{i-1}), \qquad (1)$$

where $i=1,2,\ldots,N$, m is the mass of the particle, and $\ddot{x}$ and x denote the position and instantaneous velocity, respectively. $-m\gamma\dot{x}$ describes the frictional force on the particle with the damping coefficient γ, and K measures the coupling strength of particles. The average spacing between two particles in the absence of external potentials is set to be a, which does not explicitly appear in Eq. (1). $V(x,t)$ is a periodically on-off ratchet potential:

$$V(x,t) = \begin{cases} V(x), & \text{if } t \in [nT,(n+1/2)T), \\ 0, & \text{if } t \in [(n+1/2)T,(n+1)T], \end{cases} \qquad (2)$$

where T is the flashing period. When the periodic potential is symmetric and there is no flashing on the potential, the above equations of motion describe the dynamics of the Frenkel-Kontorova model, which has been exhaustively explored to investigate numerous phenomena—for example, dislocations in crystals, charge-density waves, sliding frictions on adsorbed surfaces, flux dynamics in superconducting lines, vortices in Josephson-junction arrays and ladders, lattice heat conductions, and self-organized criticality, to name but a few [21]. In recent years this model has been extensively used to study directed transport [17,19–22]. Here we still adopt this famous model as our working system with some slight modifications. For simplicity, the periodic potential in the ON state is assumed to be a piecewisely linear sawtooth type, as shown in Fig. 1:

$$V(x) = \begin{cases} \dfrac{d}{b_1}(x-b), & nb \leqslant x < nb + b_1, \\ -\dfrac{d}{b-b_1}[x-(n+1)b], & nb + b_1 \leqslant x < (n+1)b, \end{cases} \qquad (3)$$

where d and b denote, respectively, the height and period of the substrate potential and $n=\text{int}(x/b)$. Obviously, if $b_1 = b/2$, the potential $V(x)$ is spatially symmetric; i.e., $V(-x) = V(x)$. We will focus on the case of a ratchet potential. In the overdamped limit—i.e., $\gamma \gg 1$—the inertial term $\ddot{x}$ can be neglected. In this case, we find that no net current can be observed, even if a flashing potential and coupling of particles are considered. However, if the damping coefficient γ is not so large that the dissipation rate of the kinetic energy is slow enough, then the energy from the flashing potential can well compensate for this energy dissipation. In this situation it is possible to observe a directed transport. Particularly, the presence of mutual coupling may bring competitions between different spatial length scales, which leads to complicated transport dynamics.

In the following discussions, we usually set the parameters as $N=100$, $d=1$, $b=1$, $b_1=0.1$, $a=1.7563$, $\gamma=1.0$, and $T=1$ unless specifically mentioned. Numerically the fourth-order Runge-Kutta algorithm is used and open boundary conditions are adopted. The integral steps are adjusted according the flashing time T.

III. RESULTS

A. Coupling-enhanced directed transport

The current of the directed motion can be measured by the following average drift velocity:

$$v = \frac{1}{N}\sum_{j=1}^{N} \lim_{T\to\infty} \frac{1}{T}\int_0^T \dot{x}_i(t)dt, \qquad (4)$$

where the averages include both the long-time average and the particle numbers. We further introduce the following symmetry parameter to measure the degree of the ratchet asymmetry:

$$\chi = \ln[(b-b_1)/b_1]. \qquad (5)$$

In the symmetric case, $b_1=b/2$, and thus $\chi=0$. As long as $\chi \neq 0$, the periodic potential should be a ratchet. A larger χ represents a higher asymmetry. In this paper we are interested in the deterministic transport in flashing ratchet potentials; i.e., we do not consider the effect of thermal noise. In this case, the collectiveness brought by mutual couplings among particles plays a significant role. The transport current obviously depends on various parameters, such as the coupling strength K, the flashing period T, the symmetry parameter χ, and the average spacing of particles.

In the absence of any thermal fluctuations, a finite degree of asymmetry is required for the system to produce the directed motion. In Fig. 2, the current versus the symmetry parameter for 100 particles is plotted for $K=0$ and 10.0. For both cases, it can be found that $v \neq 0$ at almost the same critical χ_c. However, they obey different scaling laws near the critical point. In the absence of mutual interactions, the current versus χ obeys a scaling as $v \propto (\chi-\chi_c)^2$, while $v \propto (\chi-\chi_c)^{1/2}$ for $K \neq 0$. This indicates a distinct difference between the single-particle and coupled cases. Moreover, it can be found that the directed current is remarkably improved for the coupled case for a moderate χ, as shown in Fig. 2. This indicates the effect of *coupling-enhanced transport.*

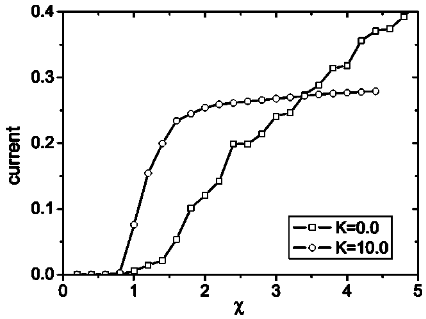

FIG. 2. The current varies with the asymmetry parameter χ for $K=0$ and $K=10$. The critical point is labeled as χ_c.

Coupling-enhanced transport has been studied previously for systems under dc forcings. Here we are interested in the effect of coupling on directed motion. In Fig. 3, the current varying against the coupling strength is plotted. It can be found that the transport against the coupling experiences a suppression-enhancement transition. For weak couplings, the directed current decreases with increasing the coupling; i.e., the transport is suppressed. This can be understood as follows. A very weak coupling—i.e., a loose connection between two particles—may introduce dispersions of phonons. One particle may hinder the directed motion of another one, and this brings forth additional dissipation; i.e., a part of energy input is dissipated by exciting phonon modes. Only when the coupling strength exceeds a threshold K_c will the current exceed the uncoupled case, and the transport then is enhanced, as shown in Fig. 3(a). In this case particles can cooperate with each other in order to produce a high-efficiency directed transport. For a very strong coupling, however, the chain behaves like a stick. In this case, the directed transport efficiency again is suppressed. In Fig. 3(b), we compute the current for stronger couplings. It can be clearly found that the current decreases with increasing coupling strength. Therefore, an optimal coupling can make the lattice moving with the highest efficiency.

B. Flashing-time-induced resonance

It is of interest to investigate the role of the flashing period T. Obviously when $T=0$, the particles get pinned by the ratchet potential even in the presence of couplings, leading to $v=0$. Here we are concerned with the deterministic case and

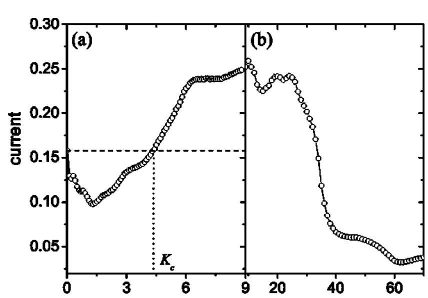

FIG. 3. The relation between the current and coupling strength K.

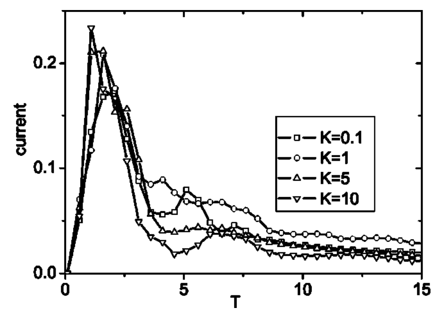

FIG. 4. The current varies with the flashing period T for $K=0.1$, 1.0, 5.0, and 10.0. Optimal switching time for an efficient transport can be observed.

try to study the dynamical mechanism of directed transport. Therefore the effect of nonequilibrium thermal noise on directed transport is beyond our focus [1]. For a very small T, a fast switch of the ratchet potential makes the particle have no time jumping to another potential well. When the switching period T is very large, the kinetic energy has been dissipated by damping before the particle gains new energy input. In both cases one cannot observe the directed current. However, for a moderate T, the appropriate match between the inertia effect and the flashing ratchet potential may lead to a nonzero current. In Fig. 4, the relation between the current and flashing period T is plotted for different coupling strengths. When $K=0$, the current exhibits some peaks, and the height of the peaks decreases with increasing T. For $K\neq 0$, it can be found that most of the peaks are smoothed and gradually suppressed with increasing K, while the highest peak is still kept and moreover enhanced for strong couplings. This indicates a distinct difference for slow and fast switches of the ratchet potential. An optimal switching time may lead to the most efficient directed transport.

C. Inertia effect: Resonances and current reversal

The inertia term in Eq. (1) plays a significant role in governing the dynamical behavior of the transport process. This can be studied by varying the damping coefficient. To study the inertia effect, one has to first consider the transport property of the single-particle case. The transport feature is closely related to the nonlinear dynamics of the system. For our underdamped system, there are mainly two types of dynamical solutions: i.e., the chaotic motion and the periodic solutions. In Fig. 5(a), the current against the damping γ is computed for the single-particle case. Two interesting phenomena can be clearly observed. First, the system has a very low directional current in most regimes of γ. These low-current regimes correspond to chaotic motions of the particle; i.e., the chaotic motion mimics the effect of a thermal noise on the system [16]. This chaotic "noise" may suppress the directed transport. On the other hand, some plateaus $v=1/4,1/3,1/2,1,\ldots$ can be clearly found. These resonant steps correspond to periodic motions. They are related to the invariance of Eq. (1) under the transformation

$$\mathbf{T}\{x(t)\}=x(t+nT)=x(t)+mb, \quad m,n\in Z, \tag{6}$$

which indicates that the particle can jump over m barriers over n switchings of the ratchet potential. Therefore the current is given as the following resonant steps:

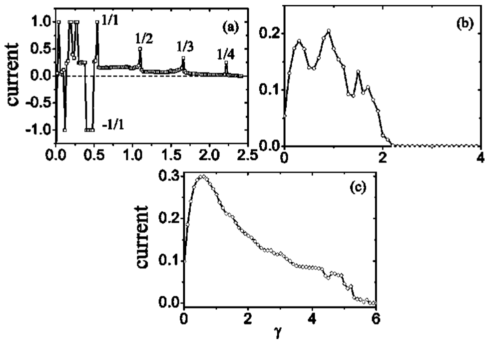

FIG. 5. The relation between the current and damping coefficient γ for (a) the single-particle case, (b) the case of two coupled particles, and (c) the case of $N=100$. $K=10$ for (b) and (c).

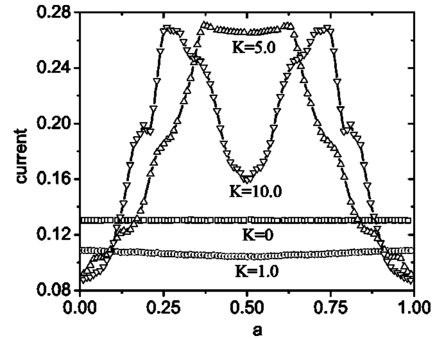

FIG. 6. The commensurate effect of the transport. Four lines for $K=0$ (squares), 1 (circles), 5 (up triangles), and 10 (down triangles) are plotted.

$$v = \frac{m}{n} v_0, \tag{7}$$

where $v_0 = b/T$. In our case $b=1$, $T=1$, and $v_0=1$. This well predicts the above resonant steps. Second, in some regimes one can find the current reversal, $v<0$; i.e., the particle may persistently move against the natural tendency of the ratchet potential. For example, for $\gamma \in (0.4, 0.48)$, a negative resonant step $v=-1$ can be observed. The phenomenon of current reversal is a very interesting issue, and it has been exhaustively studied in the presence of various thermal noises [1]. The dynamical current reversal in deterministic systems has also been studied in detail recently. Different from the pulsating ratchet type that we are discussing here, all these studies are based on the underdamped periodically rocking (tilting) ratchet models. Mateos [23] numerically showed that the origin of current reversal is related to the bifurcation from chaotic to periodic motions. Arizmendi *et al.* [24] conjectured that the mechanism of reversal is due to the crisis bifurcation, where the chaotic motion suddenly becomes periodic. Barbi and Salerno [25] argued that current reversal can even be found in parameter regimes where there is no chaotic-periodic transitions. Very recently it was shown that type-I intermittency exists when a current reversal occurs from chaotic to periodic regimes [26]. Here in our underdamped pulsating ratchet system one may find that the negative current in our case is mainly due to the existence of periodic windows. However, as shown above, a chaotic-periodic transition may not definitely give rise to a current reversal. Whether there is a similar mechanism responsible for the current reversal in pulsating ratchets is still an open problem. A detailed study of the mechanism of current reversals in underdamped flashing ratchets, as we know, has not been addressed before. This topic now is still in progress.

In the presence of coupling, it is found that current reversal can be eliminated. Figure 5(b) shows the current against the damping for $N=2$ particles. It can be found that current reversal is eliminated in all regimes of γ. Moreover, the transport can be enhanced in a much larger regime of the damping. This can be seen more clearly for the case of $N=100$, as shown in Fig. 5(c). The directed transport can be observed for $\gamma \in (0, 6.0)$. This range is much larger than that shown in Fig. 5(a). In this sense, the coupling may decrease the dissipation and enhance the transfer between the switching energy and the work of directed motion.

D. Commensurate effect

The static length a, more accurately, the frustration $\delta = a/b$, is an important parameter in determining the transport properties of the coupled system. The current depends on the choice of both the static length a and the period of the potential b. These two spatial lengths can compete and give rise to complicated commensurate effects. This has been extensively studied for the famous Frenkel-Kontorova model in relating to the ground state (Aubry's transition), the motion under dc and ac forcings, and diffusions in the presence of thermal noise [21]. In Fig. 6, the current against a (in our case b has been set to be 1) for different coupling is computed. All curves possess the following symmetries:

$$v(nb + a) = v(a), \quad v(a) = v(b - a). \tag{8}$$

These symmetries can be easily proven by analyzing the transformation invariances of Eq. (1). Obviously when $K=0$, changes in a do not affect the current. For a weak coupling $K=1.0$, it can be seen the current curve is still flat but becomes lower, especially around $a=1/2$. This indicates that the directed transport is suppressed for all a's. The situation becomes different for stronger couplings. For moderate couplings—for example, $K=5.0$—it can be found that for about $a<1/6$ or $a>5/6$, the transport is still suppressed. But for $1/6<a<5/6$, directed transport is enhanced. A plateau can be found at $3/8<a<5/8$. For a very strong coupling $K=10.0$, one can see that the current around $a=1/2$ again becomes small. On the other hand, one can still find in some regimes that the transport is enhanced. It should be noted that the relation between the current and a depends crucially on other parameters—e.g., the flashing time T, the asymmetry of the ratchet, and the damping coefficient.

IV. CONCLUSIONS

In conclusion, in this paper we investigated the deterministic inertia-induced directed transport of an elastically coupled lattice of particles in a periodically flashing ratchet potential. We find the directed transport when the asymmetry of the ratchet potential exceeds a threshold. Mutual couplings among particles may improve the transport efficiency when the coupling strength overcomes a threshold. An optimal flashing period can facilitate the transport efficiency. The inertia-induced current can be found only in a finite regime of the damping. For a single particle, current reversals and resonant transport can be observed. The coupling can eliminate the current reversal and moreover enhance the transport. We also discuss the complex commensurate effect on direct transport. Some further issues, such as the noise effect, the origin of current reversal in pulsating ratchets, coupled high-dimensional pulsating ratchets, and so on, are still under considerations and will be extensively studied.

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